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Prizewinning Essays

Second Prize

Mathematically Equivalent Formulations and Structural Realism

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The modern debate about scientific realism has evolved largely under the pressure of two opposing arguments, the no-miracles argument (NMA) and the Pessimistic Meta-Induction (PMI). The former aims to defend realism by arguing that it is the best explanation for the success of science as a whole. PMI, on the other hand, bolsters the anti-realist position by pointing to past successful theories which are now considered false, and thus arguing that we have inductive grounds to doubt the truth of our current best theories.

Structural realism (SR) is a hybrid position, popularized in recent times by Worrall (1989), which claims to defeat these arguments. In brief, it concedes that we might have good reason to be sceptical about the unobservable entities postulated by science, given the historical record of discontinuities between successive theories, but alleges that science's success is evidence that the logical and mathematical structure of our best theories must be 'latching onto' some underlying reality.

In this essay, I'll argue that SR is indeed the best of both worlds, but will mostly sidestep the well-trodden 'global' arguments mentioned above in favour of an argument related to the problem of underdetermination. Specifically, I claim that SR is compatible with the existence of mathematically equivalent formulations of physical theories in a way that standard scientific realism is not.

Every physics student learns about Newton's second law and the work-energy theorem, two distinct 'pictures' of the machinery of classical mechanics (CM). Although the former proposes the existence of vectorial forces that act upon objects, whereas the latter speaks in terms of scalar potential energies, the two formulations are mathematically equivalent. Later in her studies, the aspiring physicist will confront yet further variations in the Hamiltonian and Lagrangian formulations of CM. Although all are useful in different contexts, we might ask whether one formulation is truer – whether it refers to the fundamental constituents of nature better than its rivals.

The traditional positivist theories of scientific knowledge eschew such inquiries. On one hand, semantic instrumentalism claims that the equations in question are literally meaningless and should be viewed as fictions whose utility lies in their ability to explain sense data. In a similar spirit, reductive empiricism claims that, while the equations are meaningful, they are actually disguised propositions about sense data. For both positions, the existence of mathematically equivalent formulations doesn't present a problem: in the first case, claims about invisible forces and potential fields are not assertoric and so cannot be in conflict; in the second case, different formulations are empirically equivalent and so literally mean the same thing. But both of these anti-realist positions run afoul of the no-miracles argument. Surely the best explanation of the success of Newtonian mechanics – or any other colossally successful theory whose claims have endured many different means and modalities of investigation over a long period – is that it (at least approximately) describes some underlying reality.

Nevertheless, the would-be realist faces the problem of deciding which variation of CM is true, in the sense that its presumed entities actually refer. Without a reasonable selection criterion, one has the basis for a sceptical argument against any particular formulation the realist decides to endorse. As Jones (1991) famously aphorizes, the problem: “Realism about what?”

A rational criterion for theory selection seems unlikely. All formulations of CM are mathematically equivalent and thus empirically equivalent. (Call two theories T1 and T2. If T1 implies observation E, then, given that T2 logically implies T1, it also implies E by transitivity.) Accordingly, we might try to appeal to so-called super empirical virtues: perhaps one theory is more parsimonious or has greater practical value, for example. However, these criteria as stated fail: physicists retain all variants of CM precisely because they’re all parsimonious or practically convenient in different local contexts. Other discriminating criteria (like those suggested in Laudan 1990) similarly don’t seem to work here.

In a nutshell, this is a particularly noxious strain of the problem of strong underdetermination (summarized in Ladyman 2002 ch. 6): there exist multiple empirically equivalent theories for which no crucial experiment exists, even in principle. In the literature, underdetermination often exhibits a hypothetical character – putatively equivalent alternatives are contrived tricks (see Laudan and Leplin 1991), or are not legitimate alternatives. But the regular, almost mundane usage of mathematically equivalent theories in every branch of physics is certainly not an armchair concern.

Other classic arguments against underdetermination also seem insufficient here. For instance, following Duhem (1914), one might claim that theories are only tested against a background of auxiliary assumptions, some of which may be modified in the future. In the case of CM, since its modern successors (i.e., relativity and quantum mechanics) are not typically cast in terms of forces, we might have reason to prefer its field-theoretic formulation. The rejoinder here is that these successor theories can also be reformulated. For instance, general relativity is nowadays interpreted as postulating a non-Euclidean spacetime, thus doing away with the gravitational force entirely – but it can be equivalently interpreted as describing a Euclidean spacetime along with a universal deforming force (as discussed in Reichenbach 1928). Quantum mechanics is likewise rife with formulations (Styer 2002). In short, it is not a bug of one particular theory that it should have multiple formulations, but rather a general feature of any theory cast in the language of mathematics.

There is a sense in which our confusion arises because it is psychologically possible to form different mental pictures of the same thing, and thus mistakenly assert a difference where there is none. For example, according to the relativity of velocities, ‘Alice moves away from Bob with velocity 0.5c’ and ‘Bob moves away from Alice with velocity -0.5c' refer (ceteris paribus) to the same situation, even if we articulate them in different ways. There is no dilemma for realism here if one accepts that both 'pictures' mean: 'The Alice-Bob distance increases at a rate of 0.5c'. The problem of which is real can thus be dissolved, and one can even correct one's naïve intuition that 'Alice's velocity' and 'Bob's velocity' are independently meaningful. However, in the case of complex physical theories, the network of mathematical and logical relationships between different pictures might make such an intuitive reconciliation impossible. In such circumstances, separate pictures that are mathematically equivalent might continue to seem to speak about distinct entities whose reality needs to be decided upon.

This essay is agnostic about whether all entities are dissoluble in this way, but rather argues that the mere possibility of such a dissolution should draw us towards a general scepticism about the entities in our theories. Since mathematical manipulations can quickly multiply the number of formulations of a given theory, it seems that one is taking on considerable epistemic risk by singling out certain unobservable entities among a plethora of candidates. Even if a theory presently only has one widely-used formulation, it isn't unreasonable to expect that some new formulation might compel us to 're-picture' things at a later moment. On the other hand, mathematical structure is by definition preserved when moving between formulations, and is thus immune to this kind of sceptical argument.

This idea – scepticism about entities, confidence about structure – is the essence of structural realism. For SR, it is no problem that there are mathematically equivalent theories that postulate a distinct unobservable 'furniture' of reality. If the mathematical and logical structure of a theory are all that can be known, then we have no epistemic access to entities – they are, as Poincaré pronounced, like Kant's noumena.

In the case of CM, SR holds that the true, knowable content of the theory is in its relations and properties: If the mathematics of scalar fields adequately describes reality, then it necessarily follows that the fields in question have gradients that can be identified with forces. Asking whether forces are caused by 'real' potential fields, or whether potential fields are an abstraction of 'real' forces, is futile – and possibly confused in a similar way to the Alice-Bob dilemma given above.

Furthermore, it seems that, from a practical point of view, knowledge of entities has little bearing on a theory's success. Rather, theories like CM are successful because they possess an intricate mathematical and logical structure that allows them to express necessary connections in nature. This even allows for theories with completely fictitious entities (e.g.: Fresnel's optics) to be successful. (This was Worrall's primary reason for advocating SR in lieu of standard realism: Abandoned theories often have structure that carries over to their successors, even when their supposed entities are discarded. This explains how false theories can be successful, thus providing a response to the PMI.)

SR is not immune to criticism. For instance, I have used the word "structure" to loosely refer to the mathematical framework of a theory, but formalizing this idea is not easy, and it isn't clear that a neat distinction can be made between structure and entities (Andreas 2021). Yet it seems difficult to reconcile standard realism with the existence of mathematically equivalent formulations. Even granting that CM – or whatever other theory – is only an approximation (i.e., literally false), as long as we accept that there is some (potentially undiscovered) mathematics that does map onto the world, we are opening the floodgates to a potential torrent of formulations, each with its own zoo of entities. In view of this, epistemic modesty with regard to entities seems sensible.

Bibliography

Andreas, H., "Theoretical Terms in Science", The Stanford Encyclopedia of Philosophy (Fall 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2021/entries/theoretical-terms-science/

Duhem, P., 1914. La théorie physique son objet et sa structure, 2nd ed., Paris: Chevalier et Rivière. English Translation Philip P. Wiener, The Aim and Structure of Physical Theory, Princeton: Princeton University Press, 1954.

Jones, R., 1991. "Realism about what?", Philosophy of Science, 58

Ladyman, J., 1998. "What is structural realism?" Studies in History and Philosophy of Science, 29: 409–424

Ladyman, J., 2002. Understanding Philosophy of Science. London: Routledge.

Ladyman, J., "Structural Realism", The Stanford Encyclopedia of Philosophy (Winter 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2020/entries/structural-realism/

Laudan, L., 1981. "A confutation of convergent realism," Philosophy of Science, 48: 19–49.

Laudan, L., 1990. "Demystifying Underdetermination." In C. Wade Savage (ed.), Scientific Theories. University of Minnesota Press. pp. 267-97.

Laudan, L. and Leplin, J., 1991. "Empirical Equivalence and Underdetermination." Journal of Philosophy 88: 449-72

Reichenbach, H., 1928. The Philosophy of Space and Time, engl. transl. by M. Reichenbach and J. Freund, New York: Dover Publications

Styer, D. et al., 2002. "Nine Formulations of Quantum Mechanics." American Journal of Physics 70: 288.

Worrall, J., 1989. "Structural Realism: The Best of Both Worlds?" Dialectica, 43: 99–124.