Truth and Modality

How do we reconcile them?
Overview of talk

• The ‘truthmaking’ account of truth and modality.

• 2 problems for the truth of modal statements

1) Accounting *for* the perceived truth of modal statements – ‘Possible Worlds’ semantics

2) What are the *truthmakers* for modal statements? On what grounds do we judge a modal statement true or false? Discuss!
Truthmaking and modality

• Accept ‘truthmaker theory’ as the only ‘Rottweiler realist’ theory.

• Truth **bearers** can be sentences, statements, assertions and thoughts, all of which express **propositions**.

• Truth **makers** are ways some things in the world are

• The truthmaker of the statement 'There are 57 people in this room', if true, are the 57 people actually in this room. A proposition or statement is true if it correctly **represents** a way things actually are.

• A **modal** proposition states what is **possible**, **necessary** or **impossible**. It can also assert what **might be** or what **might have been** the case. It does **not** state what actually **is**.

• So modality creates a problem for theories of truth and truthmaking
Modal statements - some examples:

1) Pigs might fly. (F)

2) Necessarily, bachelors are male. (T)

3) David Cameron won the 2015 election (T), but he might have lost it (T).

4) I am a human being (T), but I might have been a poached egg or a football match (F).

5) The number 8 is necessarily even. (T)

6) The number of planets is necessarily even. (F)

7) There are 11 players in a football team. (T) It's possible that there might have been 13 (had the rules been set up differently) (T), but it's impossible that there could have been zero players (T).

8) No individual concrete object can be in two distinct spatiotemporal locations (No one thing can be in two places at the same time). (T)

9) Necessarily gold is the element with atomic number 79. (T)

10) Necessarily whales are mammals. (T)
Problems with modality and truth

• We have little difficulty in judging intuitively whether modal statements are true or false.

• But, given that they concern facts which do not obtain, there are two key problems with relation to their truth.

1. How to account for their truth within a logical system, so that our arguments come out valid & our theories come out true and provable.

2. The second - and deeper - question is to state why a modal statement is true or false - what is its truthmaker?
How truth values are assigned in *non-modal* logic

a) Propositional logic – key ingredients

1. **Atomic sentences (propositions)**: ‘pigs fly’; ‘bachelors are male’.
2. **The logical connectives**:
   a) Conjunction (and) \( \& \)
   b) Disjunction (or) \( \lor \)
   c) The material conditional (if...then) \( \rightarrow \)
   d) The biconditional (if and only if - iff) \( \iff \)
3. **The negation operator** \( \neg \)

Combine them as follows:

11) (Pigs fly) \( F \) \& (bachelors are male) \( T \) – FALSE.
12) (Pigs fly) \( F \) \lor (bachelors are male) \( T \) – TRUE
13) \( \neg \) (Pigs fly) \( T \) \& (bachelors are male) \( T \) – TRUE

**Key point**: all these logical operators are *truth-functional*: they affect the *truth value* of the sentences in which they occur.
b) Predicate logic - v. rough summary

Key ingredients (in addition to the earlier ones)

1) **A variable** (1\textsuperscript{st} order logic) for the subject of a proposition:
   lower-case letters \(x, y\)

2) **2 quantifiers:**
   a) the *universal* quantifier 'all', written \(\forall\)
   b) the *existential* quantifier 'some' (‘there exists at least one’), written \(\exists\).

   'Pigs fly' becomes:
   \[\forall x (x \rightarrow Fx)\] - 'For all \(x\), if \(x\) is a pig then \(x\) flies'

   'It's not the case that pigs fly' becomes:
   \[\forall x (Px \rightarrow \neg Fx)\] - 'If \(x\) is a pig, then \(x\) doesn't fly'.

**Note:** the quantifiers \(\forall\) and \(\exists\) do **NOT** affect the truth-value of the propositions in which they occur.
Classical v. modal logic

• Classical logic is designed to assign truth values to statements of how things *are*, not with how they *might be* or *might have been*.

• It can handle ‘Pigs *don’t* fly’, but it can’t handle ‘Pigs *might* fly’

• In the mid-20C a number of modal logic systems were developed to include 2 further operators:

  *'necessarily' □ known as 'box'*
  *'possibly' ◊ known as 'diamond'*

• 'Pigs might fly' becomes $\forall x \ (Px) \rightarrow ◊(Fx)$
  ('If x is a pig, then it's possible that x flies'.)
Modal logic - continued

• Necessity, possibility and impossibility are interdefinable:

• A proposition which is necessarily true (a necessary truth) is not possibly false.
  def: \[\Box p = \neg \Diamond \neg p\]

• A proposition which is possibly true (a possible truth) is not necessarily false.
  def: \[\Diamond p = \neg \Box \neg p\]

• A proposition which is actually true is possibly true
  \[p \rightarrow \Diamond p\]
  because if p weren't possible it couldn't be actual!

• The definitions and axiom above make intuitive sense. But there are problems...
Modality in a mess (1) - the problem of 'extensionality'

• 'Demodalising' two of the sentences of slide 2:
  15) The number 8 is even. T
  16) The number of planets is even. T
  (now that Pluto has been demoted!)

• Both sentences illustrate an important principle in non-modal logic: Co-referential (co-extensional) terms, when substituted, should not affect the truth value of the sentences in which they occur.

• This principle is infringed when the necessity operator is added back in:
  5) The number 8 is necessarily even.
  True. It's not possible for 8 to be an odd number! Think about it...
  6) The number of planets is necessarily even.
  False. The number of planets could have been otherwise - and was!

• The problem is that the necessity operator is not truth-functional. It does not alter the truth value of sentences in a systematic way (unlike the necessity operator: ‘Pigs fly’ – False. ¬(Pigs fly) – True.)
Modality in a mess 2 – *structure without meaning*

- The various systems of modal logic all make different basic assumptions about the interplay of necessity and possibility; their axioms and theorems grow increasingly complex.

- We can end up with long strings of iterated boxes and diamonds:
  \[-((\Box \Diamond \Box \Box \Box \Box \Diamond p) \land \neg (\Box \Diamond \Diamond p))\]

- What on earth does this *mean*?!!!

- If we apply the ‘reduction rules’ of system S5, thereby eliminating all boxes and diamonds except the final ones, we get:
  \[-((\Diamond p) \land \neg (\Diamond p))\]
  
  *It's not the case that 'possibly p' and 'not possibly p' are both true.*
  
  This is the **modal** form of a fundamental law of logic:
  `a proposition and its negation cannot both be true.`

**Thus far:** modal logic gives strange results, does not reflect everyday usage, there are several modal systems. It gives us *syntax*, hence *structure*, but no *semantics* – no clear *definitions* of $\Diamond$ and $\Box$. 

Eileen Walker     Truth and Possibility 5-9-15
‘Possible Worlds’ to the rescue?

• In the 'Possible Worlds' semantics (developed by Saul Kripke in a series of papers between 1959 and 1963) **Necessity** and **Possibility** are defined as follows:

□ P: A proposition is **necessarily** true if it is true in **all** Possible Worlds.

◊ P: A proposition is **possibly** true if it is true in **some** (at least one) Possible World.

• Technically speaking, ◊ and □ are **not** *sentential operators* like Negation (¬) but *quantifiers*, like ∀ and ∃. This is why they are **not** truth functional.
The varieties of modality - 1

• There are various ‘varieties’ of modality: logical, metaphysical, epistemic, physical, mathematical, deontic, legal... etc. All these have their own set of Possible Worlds.

• Moreover, Kripke – and others - suggest how the various systems of modal logic might map onto the varieties of modality that we actually use.

• System D, for example, maps onto deontic necessity; this concerns what one must or must not do (ought, ought not to do) – useful in ethics and the law. Key axiom: (□p→◊p)

• Ought (□p) implies can (◊p), but doesn’t imply that p actually happens. In all possible worlds, one ought to resist torturing babies, but in some worlds such resistance fails to obtain.
Varieties of modality (2)

• Deontic logic is unusual because it looks like an instance of necessity (interpreted here as a moral imperative or necessity) ($\Box p$) not implying actuality ($p$).

• If ‘$\Box p$’ is true, it is true in all possible worlds. Since all possible worlds include the actual world, we would expect $p$ to be true in the actual world. But it isn’t always, as we saw in the case of deontic modality.

• $\Box p$ implies $p$ ($\Box p \rightarrow p$) only in the ‘alethic’ modalities – metaphysical, logical and physical (nomical), so-called because necessary truth implies actual truth.

• Metaphysical and logical necessities are those to which the ‘strongest’ logical system, S5, applies. (That’s the one with the notorious axiom ‘$\diamond \Box p \rightarrow \Box p$: ‘if t $p$ is possibly necessary, then $p$ is necessary’.)
The alethic modalities - Possibility (NOT necessity) diagram

• Logical possibility - anything which does not contradict the laws of logic is possible. Anything conceivable. Very open, used by Hume and Descartes.

• Metaphysical possibility – constrained by the natures or essences of the entities in the particular modal domain (Kit Fine, Bob Hale)

• Physical possibility – constrained by the physical laws and constants of our universe.
What are the truthmakers of our modal claims?

• Although the ‘Possible Worlds’ apparatus seems to give us a way of applying truth conditions and truth values to modal propositions, it doesn’t seem to explain modality.

• If we try to say why the modal statements we looked at earlier are true or false, we realise that we must already have some presuppositions as to how modality works before arriving at a judgment using the ‘Possible Worlds’ apparatus.

• So what do we think are the truthmakers for our modal statements?

• If the truthmaker for non-modal statements is a way some things in the actual world are, then the truthmaker for a modal statement is a way some things are in some, all, or no possible world.
Applying ‘Possible Worlds’ semantics to modal propositions

1) Pigs **might** fly. (F)

2) **Necessarily**, bachelors are male. (T)

3) Although David Cameron won the 2015 election (T), he **might have lost** it (T).

4) I am a human being (T), but I **might have been** a poached egg or a football match (F).

5) The number 8 is **necessarily** even. (T)

6) The number of planets is **necessarily** even. (F)

7) There are 11 players in a football team. (T) It's **possible** that there might have been 13 (had the rules been set up differently) (T), but it's **impossible** that there could have been zero players (T).

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